Learning Hierarchical Representations in Kinematic Space

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Abstract

Learning efficient representation of data for extracting useful information to build classifiers or predictors has opened up a new avenue in machine learning called Representation Learning. Hyperbolic embeddings is one such invaluable tool developed to represent discreet, structured and symbolic data like text, graphs and trees. Embedding data in hyperbolic space has many advantages contrary to using euclidean space, as it allows for preserving the distances and other complex relationships that exists between the symbolic data. In this paper, we explore the idea of obtaining hyperbolic embeddings by studying the kinematic space, first introduced to explain the underlying connection between information theory and AdS_3/CFT_2 correspondence encountered in theoretical physics. We investigate how to learn embeddings in kinematic space, note that our aim is not to provide a formal algorithm, but adopt a theoretic centric approach to explain a possible new way of obtaining hyperbolic embeddings. We hope this work serves as ingress for future works that will allow for a consilience between machine learning and theoretical physics.

1 Introduction

The success of machine learning algorithms can be attributed to the way in which the data is represented, the performance of these algorithms can be augmented by introducing domain-specific knowledge or engineering features that maximize its performance. Representation learning algorithms [1] aim to provide a way of representing data to ensure high fidelity, useful information extraction for designing better classifiers or predictors. Representation Learning has become an invaluable tool to represent symbolic data which have inherent latent hierarchy like trees, graphs, text etc. Authors in [7] propose to learn embeddings by embedding them in the hyperbolic space, more precisely in the n-dimensional Poincaré ball model. Hyperbolic space has alluring properties and is able to preserve the complex relationships innate to the symbolic data, contrary to euclidean space. The premise of increasing the dimensions to model increasingly complex data is only natural, authors in [5] extend the work of hyperbolic embeddings on Poincaré-ball model to a Lorentz model of hyperbolic space for learning high-quality embeddings. In this work, continuing this trend, we investigate the idea of learning the embeddings in the Kinematic Space, a space of geodesics, first introduced in [8, 2], to

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explore the connection between information theory and holography, a commonly occurring theme in theoretical physics. The kinematic space is an auxiliary lorentzian manifold also defined as a space of oriented geodesics, in this paper we investigate how the kinematic space can act as an intermediary when dealing with embeddings in hyperbolic space, more specifically the hyperboloid model ($\mathbb{H}_2$), we see that geodesics in $\mathbb{H}_2$ can be thought of as point curves in the kinematic space and a metric can be defined for finding the distances between these points. Note once again, that our treatment of obtaining hierarchical embeddings in kinematic space is theoretical and we hope to support these claims through experiments in future works. Meanwhile, we hope this work will serve as a starting point to explore a consilience between deep learning and various fields like holography, conformal field theories etc. To our best of knowledge this is the first work that attempts to learn embeddings in the kinematic space framework.

2 Kinematic Space

We start with basic definitions of kinematic space, our formulation is largely inspired by [8] and [2]. Suppose we are tasked with the calculation of circumference, or in general, the length of a convex curve $\mathcal{C}$ on the euclidean space as shown in figure 1, we can use the celebrated Crofton's formula given by,

$$length = \frac{1}{4}\int_0^{2\Pi} d\theta \int_{-\infty}^{+\infty} dp n_c(\theta, p) \tag{1}$$

where, *p* gives the distance of the straight line from the origin and θ is the polar angle.

By continually changing the values of (θ, p), we can obtain a set of lines which intersect the curve, thus we can now calculate the length of the convex curve by counting the number of lines intersecting it. $n_c(\theta, p)$ gives the intersection number with the curve. This formulation provides a mapping between the convex curve and the geodesics in the kinematic space, when $\mathcal{C}$ becomes a point, we can expect a simultaneous change in the kinematic space as well, leading to what are called the point-curves. Thus, a set of geodesics passing through a single convex curve shrinked to a point has an analogue in the kinematic space called the point-curve.

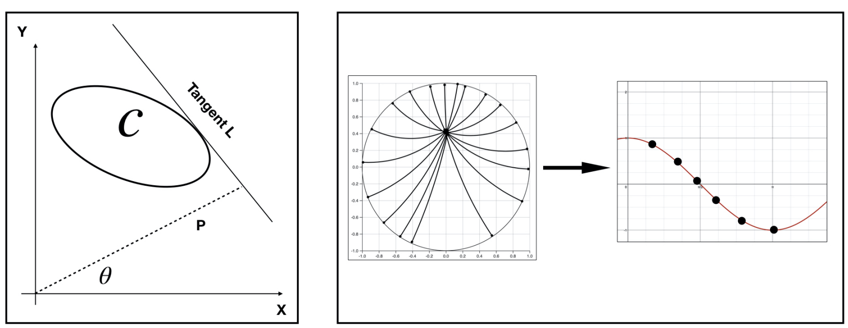

Figure 1: (Left)The length of a convex curve $\mathcal{C}$ can be calculated by counting the number of lines/geodesics intersecting it using the Crofton formula [2]. (Right)The set of geodesics passing a single point forms point-curves.

2.1 dS_2 as the Kinematic Space

We start with the hyperboloid model - $\mathbb{H}_2$, more precisely we consider the Poincaré upper half-plane model, it turns out, the kinematic space of this model is the De-sitter space (dS_2), a Lorentzian manifold, which is a pseudo-Riemannian manifold with signature (1, n-1). This model has interesting implications in theory of relativity, as it allows for including the concept of causality. Authors in [3] have demonstrated the idea of embedding graphs in the Lorentzian manifold by using Multi-dimensional scaling (MDS) algorithm. The idea of causality has an interesting application when it comes to embedding graphs which will be discussed in the next section. We follow the treatment of

[4] to show that the kinematic space of the hyperboloid model, more particularly we consider the projection of co-ordinates from the upper-half plane on to a unit disk (Poincaré disk) whose kinematic space is the deSitter space. We start with the relation -

$$X^2 + Y^2 - U^2 = 1 \tag{2}$$

We introduce the co-ordinates, U = sinhτ , X = coshτcosθ and Y = coshτsinθ, thus equation 2 becomes -

$$ds^2 = -d\tau^2 + cosh^2\tau d\theta^2 \tag{3}$$

This is the metric of the dS_2, we refer [4] for a more exhaustive approach for obtaining the metric for dS_2 space. The metric can be modified to attain the form -

$$ds^2 = \frac{dx^2 - dy^2}{dy^2} \tag{4}$$

The dS_2 space can be visualized as a hyperboloid by embedding it in a higher dimensional ambient space called the Minkowski space $\mathbb{M}$, this allows the dS_2 space to have an induced metric dictated by $\mathbb{M}$.

3 Minkowski Space and Hyperbolic Embeddings

The Minkowski space $\mathbb{M}$ is an ambient space equipped with the metric $g_{\mathbb{M}}$ for vectors *(U,V)* given by,

$$g_{\mathbb{M}}(U,V) = [u_1v_1 - u_2v_2 - u_3v_3 - ....u_nv_n] \tag{5}$$

To compute the hyperbolic embeddings, $\mathcal{E} = \{e^n_{i=1}\}$ in the kinematic space, we assume the set of symbols vary with the distance function given by,

$$d((U,V)) = arcosh(g_{\mathbb{M}}(U,V)) \tag{6}$$

The geodesics in the dS_2 are equivalent to the points formed by the intersection of a unique plane and its normal, making dS_2 a natural auxiliary geometry of $\mathbb{H}_2$. Thus, the major advantage of computing embeddings in the kinematic space is that distance between the geodesics is equivalent to calculating the distance of points. Note that work [6] also attempts to learn word embeddings using the hyperboloid model but our work highlights learning in the kinematic space.

3.1 Implications of Causality in Graph Embedding

The points in kinematic space can be classified as time-like, space-like and light-like separated, these classifications have special importance when studying the theory of relativity, but in our case the idea of points being time-like, space-like and light-like could assist in embedding the graphs. Two points are said to be time-like separated if the geodesics formed by these points are contained in one another, they are space-like if neither one contains the other and finally light-like implies they both share a common end-point. Thus, the points in the kinematic space form a *Causal Set*. This can be characterized by a metric as outlined in [8]

$$ds^2 = \frac{\partial S^2(u,v)}{\partial u \partial v} dudv \tag{7}$$

which gives the kinematic space a causal structure. This idea is useful because most of the real-world graphs like citation networks, family trees etc share some sort of causal relationship [3], this could, in principle, help in application like link-prediction in graphs, where we are expected to ascertain the probability of links between the nodes.

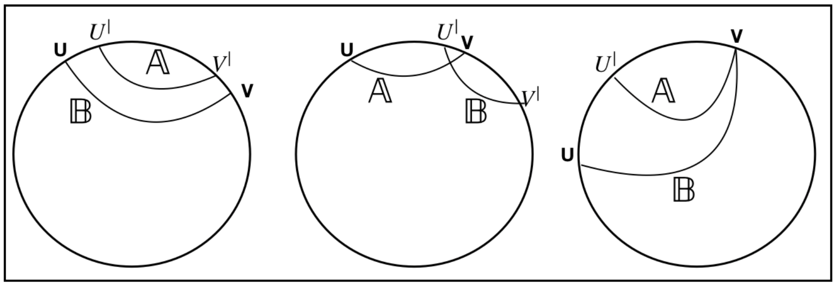

Figure 2: The visualization of time-like, space-like and light-like geodesics.

4 Conclusion

In this paper, we have explored the idea of obtaining hyperbolic embeddings by transforming our embedding space into kinematic space. We have provided a mathematical treatment of obtaining said embeddings. We hope to provide more experimental results in our future works, meanwhile we hope this work will serve as a starting point to explore possible conjunction of machine learning and theoretical physics.

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